

Lesson 1: Simplifying Square Roots and Rationalizing

Example Simplify $\sqrt{108}$.

First note that $\sqrt{108}$ is not a whole number. It is slightly more than $\sqrt{100}$ which is 10. A calculator gives $\sqrt{108} \approx 10.39230485$. This is only an approximate answer. The exact square root of 108 cannot be expressed as a terminating or repeating decimal. It is irrational, which means that the decimal expansion is non-terminating and non-repeating.

Solution 1

Idea The “perfect squares” are the squares of integers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, These are the numbers whose square roots are integers. Find the largest perfect square that is a factor of 108. Checking the list, we find that it is 36.

$$\sqrt{108} = \sqrt{36 \cdot 3} = \sqrt{36} \sqrt{3} = 6\sqrt{3}$$

Solution 2

Idea It is usually time-consuming to find the *largest* perfect square that is a factor of a given number. So, we will find *any* perfect square that divides 108. We notice immediately that 108 is divisible by 4.

$$\sqrt{108} = \sqrt{4 \cdot 27} = \sqrt{4} \sqrt{27} = 2\sqrt{27}$$

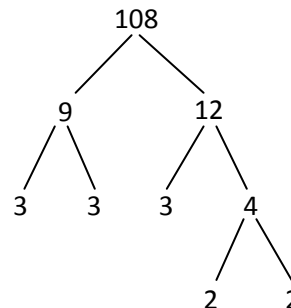
Note that we are not done because 27 is divisible by another perfect square, namely 9.

$$\sqrt{108} = \sqrt{4 \cdot 27} = \sqrt{4} \sqrt{27} = 2\sqrt{27} = 2\sqrt{9 \cdot 3} = 2\sqrt{9} \sqrt{3} = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$$

Solution 3

Idea We factor 108 into prime factors as in the diagram on the side.

$$\sqrt{108} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \sqrt{2 \cdot 2} \sqrt{3 \cdot 3} \sqrt{3} = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$$



Example Rationalize the denominator: $\frac{3}{\sqrt{6}}$.

Note that in the fraction, the denominator is an irrational number. Our goal is to find an equal fraction whose denominator is rational.

Solution

Idea Multiplying the fraction by 1 does not change its value.

$$\frac{3}{\sqrt{6}} = \frac{3}{\sqrt{6}} \cdot 1 = \frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{\sqrt{6}\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$$

We can check our answer on a calculator. The value of $\frac{3}{\sqrt{6}}$ is about 1.224744871. So is the value of $\frac{\sqrt{6}}{2}$.

Example Rationalize the numerator: $\frac{\sqrt{3}}{3}$.

Solution

$$\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3} \cdot 1 = \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}}{3\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$