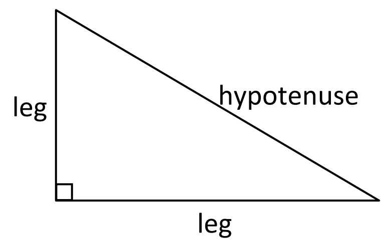
# Lesson 2: The Pythagorean Theorem

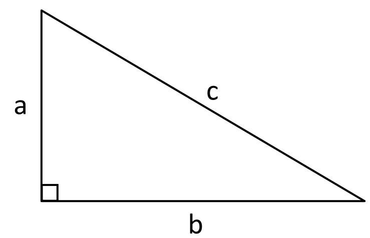
A right triangle is any triangle containing a right (90 degree) angle. The side opposite the right angle is called the **hypotenuse** and the other two sides are the **legs**.




The Pythagorean Theorem states that in any right triangle

$$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$$

Another way to state the Pythagorean Theorem is to label the hypotenuse "c" and the two legs "a" and "b".

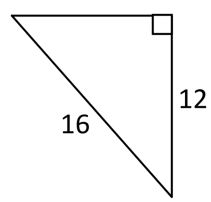



Using this notation, the Pythagorean Theorem is

$$a^2 + b^2 = c^2$$

The Pythagorean Theorem can be used to find the missing side of a right triangle when two sides are known.

**<u>Example</u>**
Find the missing side in the given right triangle.




**<u>Solution</u>** We call the missing side *a*. Then

$$a^2 + 12^2 = 16^2$$

$$a^2 + 144 = 256$$

$$a^2 = 112$$

$$a = \sqrt{112} = \sqrt{4 \cdot 28} = \sqrt{4}\sqrt{28} = 2\sqrt{28} = 2\sqrt{4 \cdot 7} = 2\sqrt{4}\sqrt{7} = 2 \cdot 2\sqrt{7} = 4\sqrt{7}$$

**Example** The legs of a right triangle are 2 and $2\sqrt{3}.$ Find the hypotenuse.

**Solution** We let *c* denote the length of the hypotenuse. Then

$$c^2 = 2^2 + \left(2\sqrt{3}\right)^2$$
$$c^2 = 4 + 2^2\sqrt{3}^2$$
$$c^2 = 4 + 4\cdot 3$$
$$c^2 = 16$$
$$c = \sqrt{16} = 4$$

**Pythagorean Triples**

Note that $3^2 + 4^2 = 5^2$ (both sides evaluate to 25). So, there is a right triangle with sides of lengths 3, 4, and 5. The triple of numbers 3-4-5 is called a **Pythagorean triple**. Memorize it. This will provide you a shortcut that will avoid using the Pythagorean theorem.

If there is a right triangle with legs of 3 and 4, then you can instantly recall the hypotenuse is 5.

If there is a right triangle with a leg of 3 and hypotenuse of 5, then the missing leg is 4.

**Warning:** If there is a right triangle with legs of 3 and 5, then the hypotenuse is not 4. (The hypotenuse must be the longest side.) You should check that the hypotenuse is $\sqrt{34}.$

**Common Pythagorean Triples**

The list of Pythagorean triples is endless. Here are some of the most common:

3-4-5
5-12-13
7-24-25
8-15-17
9-40-41

Memorize at least the first two so that we can do certain problems in class more quickly. (Note that you should check, using the Pythagorean theorem, that these really are Pythagorean triples.)

**More Pythagorean Triples**

If *a*-*b*-*c* is a Pythagorean triple and *m* is a positive integer then *ma*-*mb*-*mc* is a Pythagorean triple as well. For example, since 3-4-5 is a Pythagorean triple, we can multiply the numbers by a positive integer such as 7 to get a new Pythagorean triple: 21-28-35.

**Example** If the legs of a right triangle are 10 and 24, what is the hypotenuse?

**Solution** To check for a Pythagorean triple, divide 10 and 24 by their greatest common factor, 2. We get 5 and 12. This reminds us of 5-12-13. Multiplying by 2 we get 10-24-26. So, the hypotenuse is 26.