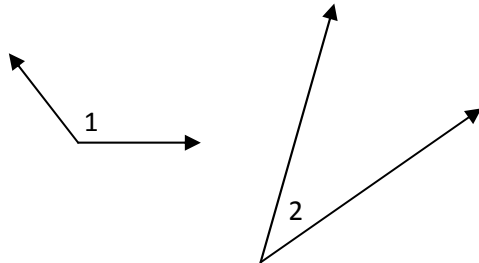


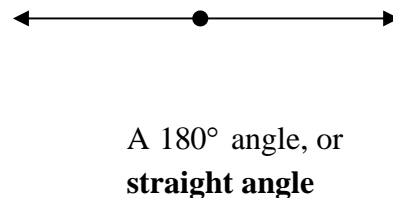
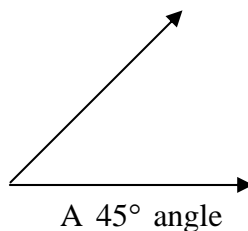
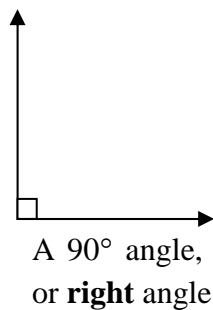
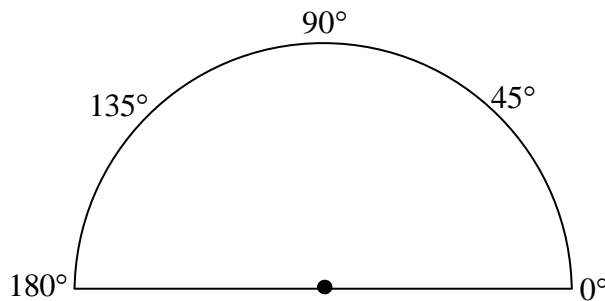
## Lesson 3: Angles

### Measuring Angles

- The wider the opening between the sides, the larger the measure of an angle.



- Here, angle 1 has a larger measure than angle 2.
- Note that the length of the sides drawn has no bearing on the measure of the angle.
- You can think of the measure of an angle as a measure of how much one side would have to be **rotated** to reach the other side.
- One unit of angle measurement is the **degree**. Every angle has a measure greater than 0 degrees ( $0^\circ$ ) and less than or equal to  $180^\circ$ .



An **acute** angle is any angle whose measure is less than 90 degrees.

An **obtuse** angle is any angle whose measure is more than 90 degrees.

The **complement** of an angle is equal to 90 degrees minus the angle.

The **supplement** of an angle is equal to 180 degrees minus the angle.

**Example** What is the complement of  $42^\circ$ ?

**Solution**  $90^\circ - 42^\circ = 48^\circ$

**Degrees, Minutes, and Seconds (DMS units)**

- 60 minutes =  $60' = 1^\circ$
- 60 seconds =  $60'' = 1'$

**Example** Change to degrees:  $32^\circ 51' 9''$ .

**Idea** We change seconds to minutes by dividing by 60. Then we change minutes to degrees by dividing by 60.

**Solution**

$$32^\circ 51' 9'' = 32^\circ + 51' + (9/60)' = 32^\circ + 51.15' = 32^\circ + (51.15/60)^\circ = 32.8525^\circ$$

**Example** Change to DMS units:  $113.1015^\circ$ .

**Idea** We change the  $0.1015^\circ$  to minutes by multiplying by 60. Then we change any fraction of a minute to seconds by multiplying by 60.

**Solution**

$$113.1015^\circ = 113^\circ + (0.1015 \cdot 60)' = 113^\circ 6.09' = 113^\circ 6' + (0.09 \cdot 60)'' = 113^\circ 6' 5''$$

(We rounded to the nearest second.)

**Example** What is the complement of  $42^\circ 45' 50''$ ?

**Idea** It is best to do this entirely in DMS units rather than convert to degrees. We will subtract the given angle measurement from  $90^\circ 0' 0''$ . To do this we will have to “borrow.” Note that  $90^\circ 0' 0'' = 89^\circ 60' 0'' = 89^\circ 59' 60''$ .

**Solution**

$$\begin{array}{r} 89^\circ \ 59' \ 60'' \\ 42^\circ \ 45' \ 50'' \\ \hline 47^\circ \ 14' \ 10'' \end{array}$$

## The Angles of a Triangle

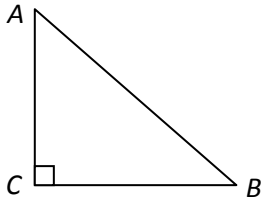
The three angles of any triangle add up to  $180^\circ$ . We can use this fact to find a missing angle in a triangle when two are given.

**Example** In  $\triangle ABC$ ,  $\angle A = 40^\circ$  and  $\angle B = 60^\circ$ . What is  $\angle C$ ?

**Solution**  $\angle C = 180^\circ - (40^\circ + 60^\circ) = 180^\circ - 100^\circ = 80^\circ$ .

Note that at most one angle of a triangle can be obtuse (since two obtuse angles add up to more than 180 degrees). Also, in a right triangle, the right angle is the largest angle (since there cannot be two angles each greater than or equal to 90 degrees).

**Example** What is the relationship between angles  $A$  and  $B$  in the right triangle below?



**Solution**

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$\angle A + \angle B = 90^\circ$$

So, angles  $A$  and  $B$  are complementary.