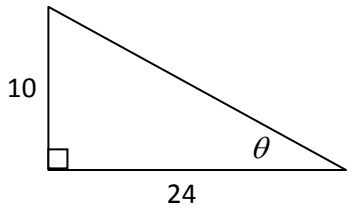
## Lesson 5: The Six Trigonometric Functions—Worksheet

Memorize the definitions of the six trigonometric functions, then solve the following problems without looking up the definitions.

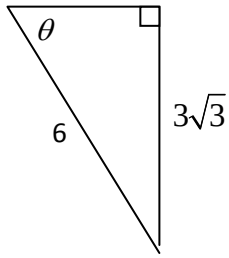
**1.** Find the values of all six trigonometric functions of $\theta$ in the following triangle.




First we find the missing side. 5-12-13 is a Pythagorean triple. Multiplying it by 2 gives 10-24-26. So, the missing hypotenuse is 26. Note that the 10 is opposite $\theta$ and the 24 is the adjacent side. So, we have:

$$\sin(\theta)=\frac{10}{26}=\frac{5}{13} \qquad \csc(\theta)=\frac{26}{10}=\frac{13}{5}$$
$$\cos(\theta)=\frac{24}{26}=\frac{12}{13} \qquad \sec(\theta)=\frac{26}{24}=\frac{13}{12}$$
$$\tan(\theta)=\frac{10}{24}=\frac{5}{12} \qquad \cot(\theta)=\frac{24}{10}=\frac{12}{5}$$

**2.** Find the values of all six trigonometric functions of $\theta$ in the following triangle.



First, we find the missing side. Call it *a*. Then

$$a^2 + \left(3\sqrt{3}\right)^2 = 6^2$$

$$a^2 + 3^2 \cdot \sqrt{3}^2 = 36$$

$$a^2 + 9 \cdot 3 = 36$$

$$a^2 + 27 = 36$$

$$a^2 = 9$$

$$a = \sqrt{9} = 3$$

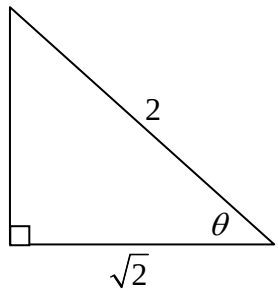
Note that the side opposite $\theta$ is $3\sqrt{3}$ and the adjacent side is 3. So,

$$\sin(\theta) = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} \qquad \csc(\theta) = \frac{6}{3\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos(\theta) = \frac{3}{6} = \frac{1}{2} \qquad \sec(\theta) = \frac{6}{3} = 2$$

$$\tan(\theta) = \frac{3\sqrt{3}}{3} = \sqrt{3} \qquad \cot(\theta) = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

**3.** $\theta$ is an acute angle in a right triangle. If $\cos(\theta) = \frac{\sqrt{2}}{2}$, then what are the values of the other five trigonometric functions of $\theta$? (Hint: draw the triangle and label its sides.)




Note that we've labeled the adjacent side $\sqrt{2}$ and the hypotenuse 2 so that $\cos(\theta) = \frac{\sqrt{2}}{2}$. We use the Pythagorean Theorem to find the missing side. Call it *a*. Then

$$a^2 + \sqrt{2}^2 = 2^2$$

$$a^2 + 2 = 4$$

$$a^2 = 2$$

$$a = \sqrt{2}$$

Now, we have

$$\sin(\theta)=\frac{\sqrt{2}}{2} \qquad \csc(\theta)=\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

$$\cos(\theta)=\frac{\sqrt{2}}{2} \qquad \sec(\theta)=\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

$$\tan(\theta)=\frac{\sqrt{2}}{\sqrt{2}}=1 \qquad \cot(\theta)=\frac{\sqrt{2}}{\sqrt{2}}=1$$